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A simple method of tuning PID controllers for stable and unstable FOPTD systems

R. Padma Sree, M. N. Srinivas, M. Chidambaram*

Department of Chemical Engineering, Indian Institute of Technology, Madras, Chennai 600036, India

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Abstract

A simple method is proposed to design PI/PID controllers for stable first-order plus time delay (FOPTD) systems. The method is based on matching the coefficient of corresponding powers of s in the numerator and that in the denominator of the closed loop transfer function for a servo problem. This method gives simple equations for the controller settings in terms of the FOPTD model parameters. Simulation results show that the method gives a similar response as that of Ziegler–Nichols (Z–N) method and better response than that of IMC method. Controllers are also designed by using two tuning parameters and the performance is best when compared to that of Z–N [ASME Trans. 64 (1942) 759] and Abbas [ISA Trans. 36 (1997) 183]. The controller settings give a robust performance for uncertainty in the process model parameters. The method is also extended to design PI/PID controllers for an unstable FOPTD system. Simulation results show that the present method gives improved performances: (i) for PID controllers over that of the controllers designed by Huang and Chen [J. Chem. Eng. Jpn. 32 (1999) 579], IMC method and that proposed by Visioli [IEE Proc. CTA 148 (2001) 180] and (ii) for PI controllers over the method of Jung et al. [J. Process Contr. 9 (1999) 265]. Theoretical analysis of stability and robustness of the proposed controller are also provided.

Keywords: PID controller; Unstable systems; FOPTD model; Tuning

1. Introduction

For the purposes of designing controllers, the dynamics of many processes can be described adequately by a first-order plus time delay (FOPTD) model. Methods for tuning PID controllers for such models are based on stability analysis (Cohen & Coon, 1953; Ziegler & Nichols, 1942), synthesis method (Smith & Corripio, 1985), constant open loop transfer function method (Haalman, 1965), pole placement method (Clement & Chidambaram, 1997a, 1997b) and internal model controller (Rivera, Morari, and Skogetad, 1986; Wang, Hang, and Yang, 2001). An excellent review of the work reported on the design of PID controllers is given by Astrom and Hagglund (1995).

The design of PID controllers for unstable FOPTD model has attracted attention recently (Chidambaram, 1997). The performances specifications that are normally obtained for stable FOPTD model cannot be obtained for unstable systems. The methods for designing PID controllers for unstable FOPTD systems are given by the modified Ziegler–Nichols (Z–N) method (DePaor & O'Malley, 1989; Ho & Xu, 1998; Venkatasubramaniam & Chidambaram, 1994), IMC method (Marchetti, Scali, & Lewin, 2001; Rotstein & Lewin, 1991), pole placement method (Clement & Chidambaram, 1997a, 1997b), optimization method (Cheng & Hwang, 1998; Manoj & Chidambaram, 2001; Visioli, 2001), two degrees of freedom method (Huang and Chen, 1997, 1999) and synthesis method (Chandrashekar, Padmasree, and Chidambaram, 2002; Jung, Song, and Hyun, 1999). In many of these methods, one or two adjustable parameters are used to calculate the PID settings.

In all the above methods, the design procedure is somewhat complicated. In the present work, a simple method is proposed to design PI and PID controllers for both the stable and unstable FOPTD systems. The equations for the controller settings are simple in terms of the model parameters.

* Corresponding author.
E-mail address: chidam@iitm.ac.in (M. Chidambaram).

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Nomenclature

k_1, k_2, k_3	defined by Eqs. (3)–(5), respectively
k_c	controller gain
k_p	process gain
q	$=s\tau$
s	Laplace variable
u	manipulated variable
y	output
y_r	set point value for y

Greek letters

α	tuning parameter
α_1	tuning parameter
α_2	$=\beta\tau_1$
β	tuning parameter
ε	τ_d/τ
τ	time constant of the system
τ_d	time delay
τ_D	derivative time
τ_I	integral time

2. The proposed method

Let us consider a FOPTD system $k_p \exp(-\tau_d s)/(\tau s \pm 1)$, with '+' sign for stable systems and '−' sign for unstable systems. Let us consider a PID controller.

The PID control law is given by

$$\frac{u(s)}{e(s)} = k_c\left[1 + \frac{1}{\tau_I s} + \tau_D s\right] \tag{1}$$

where u is manipulated variable and e is error.

The closed loop transfer function relating the output variable (y) to the set point (y_r) is given by

$$\frac{y(q)}{y_r(q)} = [k_1 q + k_2 + k_3 q^2] \times \left[\frac{\exp(-\varepsilon q)}{(s \pm 1)q + (k_1 q + k_2 + k_3 q^2)\exp(-\varepsilon q)}\right] \tag{2}$$

where

$$k_1 = k_c k_p \tag{3}$$

$$k_2 = \frac{k_1}{\tau_I/\tau} \tag{4}$$

$$k_3 = k_1(\tau_D/\tau) \tag{5}$$

$$\varepsilon = \frac{\tau_d}{\tau} \tag{6}$$

$$q = s\tau \tag{7}$$

The above equation can be written as

$$\frac{y(q)}{y_r(q)} = [k_1 q + k_2 + k_3 q^2]e^{0.5\varepsilon q} \times \frac{e^{-\varepsilon q}}{(q \pm 1)qe^{0.5\varepsilon q} + (k_1 q + k_2 + k_3 q^2)e^{-0.5\varepsilon q}} \tag{8}$$

Let us consider the numerator and the denominator terms of Eq. (8) using the Taylor series expansion for $e^{0.5\varepsilon q}$ and $e^{-0.5\varepsilon q}$. We shall remove the $e^{-\varepsilon q}$ term in the numerator for further analysis, since this will only shift the corresponding time axis. With this approach, the order of the numerator is the same as that of the denominator. The coefficient of the constant term (coefficient of q^0) of the numerator is already equal to that of the denominator because of the presence of the integral action. Let us first consider open loop stable systems. Since the objective of the controller is to make $y/y_r = 1$, let us equate the coefficient of the corresponding powers of q of the numerator with that of the denominator. On equating the powers of q, we get

$$k_2 = \frac{1}{\varepsilon} \tag{9}$$

On equating the coefficient of q^2 of the numerator and the denominator terms, we get

$$k_1 = \frac{1}{\varepsilon} + 0.5 \tag{10}$$

Similarly on equating the coefficient of q^3, we get

$$2k_3 = 1 + \tfrac{1}{6}\varepsilon \tag{11}$$

On rearranging all the above three equations, we get the following simple equations for the PID controller settings in terms of the model parameters:

$$k_c k_p = \frac{\tau}{\tau_d} + 0.5 \tag{12}$$

$$\tau_I = \tau + 0.5\tau_d \tag{13}$$

$$\tau_D = \frac{0.5\tau_d(\tau + 0.1667\tau_d)}{\tau + 0.5\tau_d} \tag{14}$$

Eq. (12) is derived using Eqs. (3), (6) and (10). Eq. (13) is derived using Eqs. (4), (6), (9) and (12). Eq. (13) is derived using Eqs. (5), (9), (11) and (12).

3. Simulation results for stable systems

Let us consider a stable FOPTD model with $k_p = 1$, $\tau = 1$ and $\tau_d = 0.5$. The PID controller settings by the present method are $k_c = 2.5$, $\tau_I = 1.25$ and $\tau_D = 0.2167$. The PID settings by the open loop Z–N method are $k_c = 2.4$, $\tau_I = 1$ and $\tau_D = 0.25$. The PID settings by the IMC method (Wang et al., 2001) are: $k_c = 1.7137$, $\tau_I = 1.0154$, $\tau_D = 0.1669$ and filter time constant $\tau_f = 0.01856$. Fig. 1 shows the comparisons of the servo response of the present method with

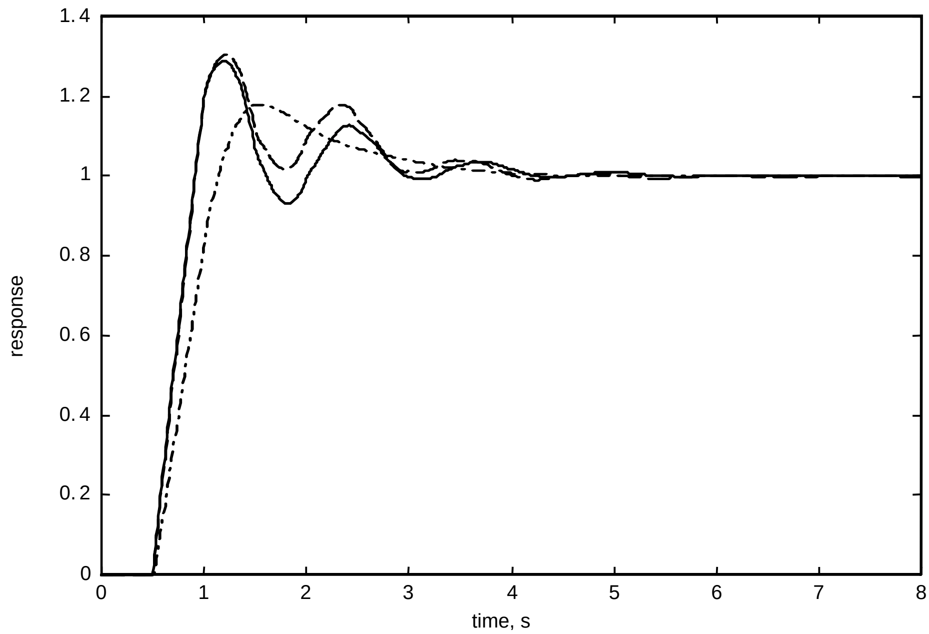

Fig. 1. Comparison of servo responses for stable systems: $k_p = 1$, $\tau_d = 0.5$, $\tau = 1$ with PID controller. Solid line: present method; dotted line: IMC method; dashed line: Z–N method.

those of the other two methods. A similar performance is obtained for the present method. Fig. 2 shows the performance for the regulatory problem (with the load transfer function same as that of the process). A response similar to that of Z–N method is obtained. Though an analytical derivation for the Z–N method is available (Chidambaram, 1998), the procedure is slightly complicated when compared to that of the present method. The IMC method proposed by Wang et al. (2001) is based on numerical optimization method. The present method is a simple one and the resulting tuning formulae are also simple ones. Table 1 gives the comparisons of ISE, IAE and ITAE for these three methods. The present method is better than that of IMC. The robustness of the controller is evaluated by perturbing the time delay as 0.6 in the process whereas the controller settings used are for $\tau_d = 0.5$. Fig. 3 shows the responses. Table 2 gives the comparisons of ISE for these methods under parameter uncertainty for a servo problem and separately for a regulatory

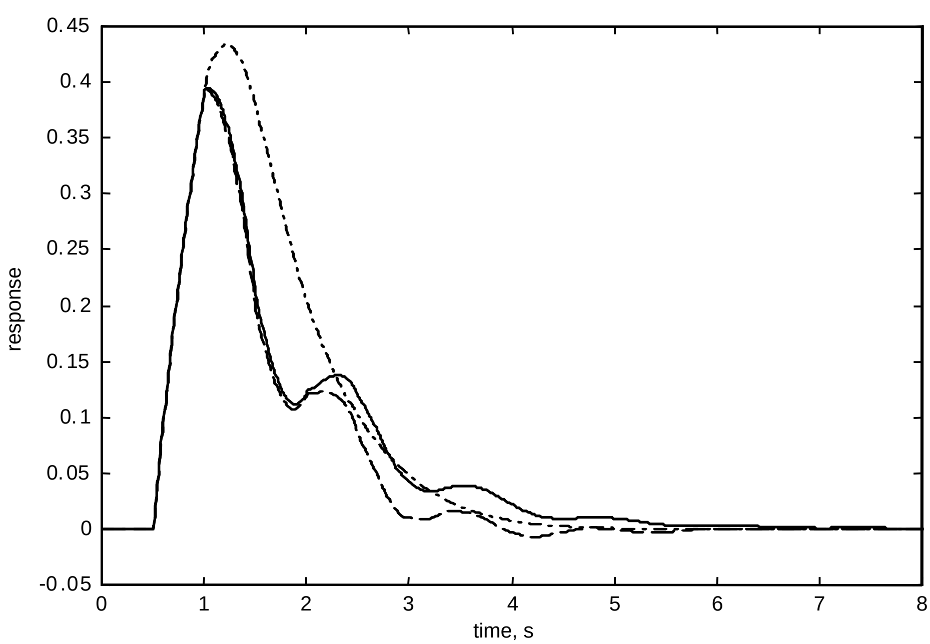

Fig. 2. Comparison of regulatory responses for stable systems: $k_p = 1$, $\tau_d = 0.5$, $\tau = 1$ with PID controller. Legend: as in Fig. 1.

Table 1
ISE, IAE and ITAE values for stable system ($e^{-0.5s}/(s+1)$) with PID controller

Method	Servo problem			Regulatory problem		
	ISE	IAE	ITAE	ISE	IAE	ITAE
Present[a]	0.6792	0.955	0.746	0.1101	0.4997	0.8776
Present[b]	0.7573	1.119	0.9604	0.0967	0.367	0.5038
Z–N	0.696	1.016	0.863	0.1009	0.4273	0.6457
IMC	0.7434	1.038	0.743	0.1732	0.5928	0.9498

IMC: Wang et al. (2001).
[a] No tuning parameter.
[b] Two tuning parameters.

problem. Similar robust performances are also obtained for uncertainty in process gain and separately in time constant (refer to Fig. 3). The performances under model parameter uncertainty are better than that of IMC and close to that of

Table 2
ISE value comparisons for stable systems with PID controller under parameter uncertainty (parameters for controller design: $k_p = 1$, $\tau_d = 0.5$, $\tau = 1.0$)

Method	Servo			Regulatory		
	ISE values for uncertainty in			ISE values for uncertainty in		
	20% k_p	20% τ	20% L	20% k_p	20% τ	20% L
Present[a]	0.7544	0.6906	0.8933	0.1328	0.1057	0.137
Present[b]	0.8212	0.7871	1.0611	0.0947	0.1151	0.091
Z–N	0.7612	0.7180	0.9064	0.1227	0.0970	0.128
IMC	0.7375	0.7884	0.8986	0.1980	0.1701	0.205

Z–N: Ziegler–Nichols method.
[a] No tuning parameter.
[b] Two tuning parameters.

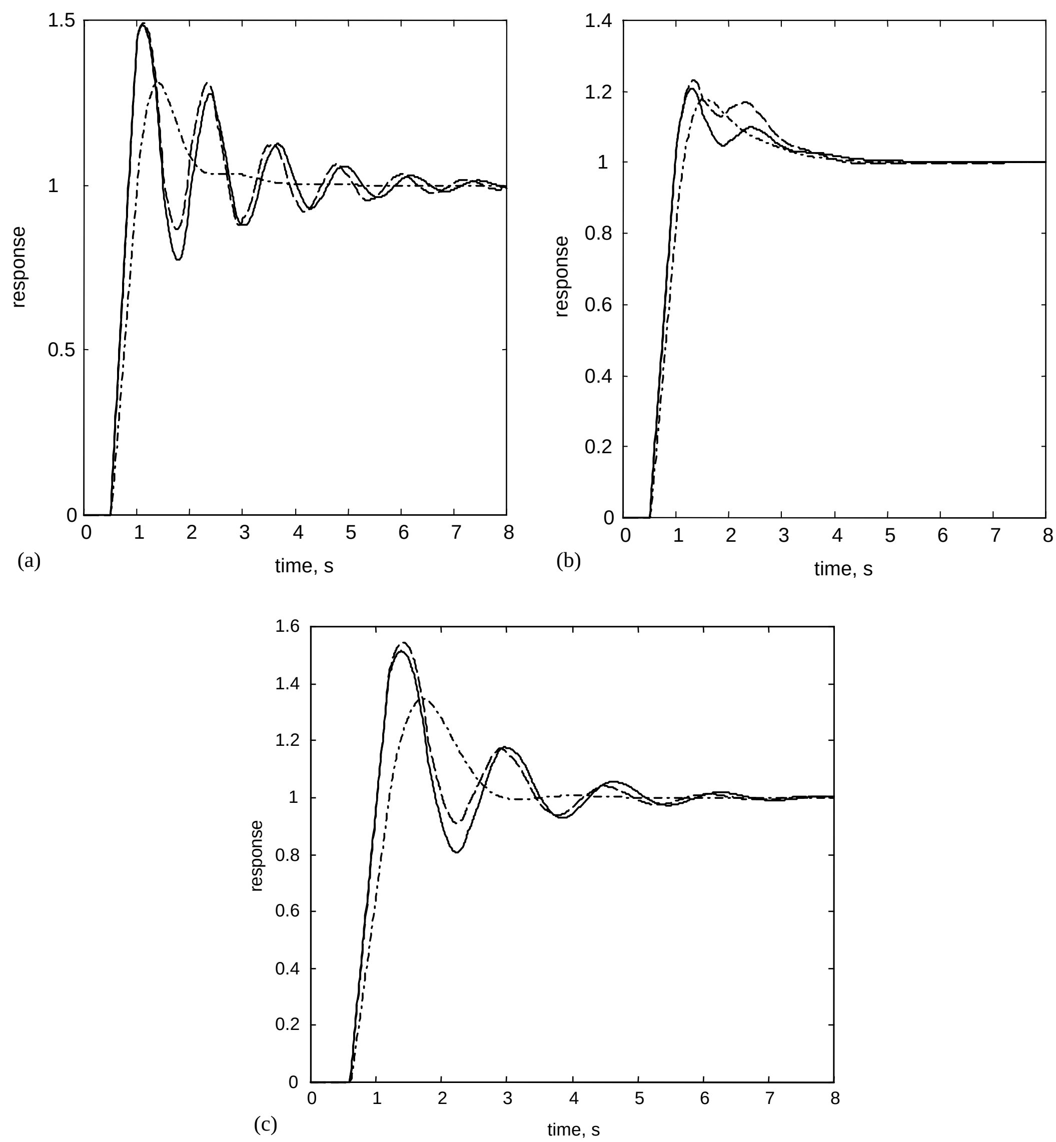

Fig. 3. Servo responses under parametric uncertainty (with PID controller): (a) uncertainty in k_p; (b) uncertainty in τ; (c) uncertainty in τ_d.

the Z–N method. As stated earlier, the simplicity in deriving the controller settings is another plus point of the present method.

4. Controller design for unstable systems

In this known that the performance specifications similar to stable systems cannot be met for the unstable systems. The overshoot and settling time are larger for unstable systems (Chidambaram, 1997). Hence, we cannot force $y(q)/y_r(q)$ in Eq. (8) as 1 by the method discussed in the earlier section. If we equate $y(q)/y_r(q)$ to 1, then it can be checked by the above procedure (similar to Eqs. (12)–(14)) that the integral time becomes negative. Such settings make the closed loop system unstable even for a small change in the system parameters. To avoid this problem, we have to make each of the numerator term of Eq. (8) (except that of the coefficient of q^0) equals to α times that of the corresponding denominator term. By doing so, we get the following set of linear algebraic equations for the PID controller settings:

$$\begin{bmatrix} 1-\alpha & 0.5(1+\alpha)\varepsilon & 0 \\ 0.5\varepsilon(1+\alpha) & 0.125\varepsilon^2(1-\alpha) & 1-\alpha \\ 0.25\varepsilon(1-\alpha) & \frac{0.25}{6}\alpha\varepsilon^2 & 1+\alpha \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} -\alpha \\ \alpha(1-0.5\varepsilon) \\ \alpha(1-0.25\varepsilon) \end{bmatrix} \quad (15)$$

Since for the larger value of ε, the overshoot is larger (Chidambaram, 1997; DePaor & O'Malley, 1989), we have to assume correspondingly a larger value for the parameter α. Here, α is a tuning parameter whose value is to be greater than 1. The suitable value of α is found out by simulation of the process model with the PID controller. Table 3 gives the value of α and the corresponding PID settings obtained. The following empirical equations are fitted for the resulting PID settings:

Table 3
PID controller settings for unstable FOPTD systems using one tuning parameter (Eq. (15))

S. no.	ε	α	$k_c k_p$	τ_I/τ	τ_D/τ
1	0.01	1.02	101.489	1.0151	0.005
2	0.025	1.05	41.4729	1.0382	0.0124
3	0.05	1.1	21.4459	1.0779	0.0246
4	0.1	1.2	11.3918	1.1620	0.0486
5	0.2	1.3	5.8451	2.9643	0.0978
6	0.3	1.4	4.1023	3.8068	0.1461
7	0.4	1.6	3.1939	5.25	0.1949
8	0.5	1.75	2.62	8.375	0.2450
9	0.6	1.95	2.27	9.75	0.2938
10	0.7	2.18	2.018	11.15	0.3427
11	0.8	2.43	1.8167	14.85	0.3928
12	0.9	2.71	1.6517	24.09	0.4440

$\varepsilon = \tau_d/\tau$.

$$\begin{aligned} &k_c k_p = 1.4183\varepsilon^{-0.9147} && \text{for } 0.01 \le \varepsilon \le 0.9 \\ &\frac{\tau_I}{\tau} = 16.327\varepsilon^2 + 5.5778\varepsilon + 0.8158 && \text{for } 0.01 \le \varepsilon \le 0.6 \\ &\frac{\tau_I}{\tau} = 196\varepsilon^2 - 247.28\varepsilon + 87.72 && \text{for } 0.6 \le \varepsilon \le 0.9 \\ &\frac{\tau_D}{\tau} = 0.4917\varepsilon && \text{for } 0.01 \le \varepsilon \le 0.9 \end{aligned} \quad (16)$$

The R^2 values (regression coefficients) for the above equations are respectively 0.9991, 0.988, 0.998 and 1.

5. Simulation results for unstable systems

Let us consider an unstable FOPTD system with $k_p = 1, \tau = 1$ and $\tau_d = 0.5$. The PID settings by the present method are given by $k_c = 2.62$, $\tau_I = 8.375$ and $\tau_D = 0.245$. For performance comparison, we use the pole placement method (Clement & Chidambaram, 1997a, 1997b) and that proposed by Huang and Chen (1999). The settings by the pole placement method are $k_c = 1.4$, $\tau_I = 5.688$ and $\tau_D = 0.341$ and the settings by Huang and Chen (1999) are: $k_c = 2.142$, $\tau_I = 2.9087$ and $\tau_D = 0.1603$. Fig. 4 shows the servo response for these settings. Fig. 5 shows the responses for the regulatory problems. The response is better than that of the pole placement method for both the servo and regulatory problems. The overshoot is lesser for the present method than that of Huang and Chen. However, it is found by simulation that response by the present method is inferior to that of the IMC method and that by Visioli (2001). Further controller settings cannot be obtained for $\tau_d > 0.9$. Hence, there is a need to improve the present method.

Hence, it is decided to equate the coefficient of power of q to α_1 times that of the denominator whereas the coefficient of q^2 and q^3 is set to α_2 ($=\beta\alpha_1$) times that of the denominator. There are two tuning parameters: α_1 and β. Similar to the steps given in the previous section, we get the following linear algebraic equations for the PID settings as:

$$\begin{bmatrix} 1-\alpha_1 & 0.5(1+\alpha_1)\varepsilon & 0 \\ 0.5\varepsilon(1+\alpha_2) & 0.125\varepsilon^2(1-\alpha_2) & 1-\alpha_2 \\ 0.25\varepsilon(1-\alpha_2) & \frac{0.25}{6}(1+\alpha_2)\varepsilon^2 & 1+\alpha_2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} -\alpha_1 \\ \alpha_2(1-0.5\varepsilon) \\ \alpha_2(1-0.25\varepsilon) \end{bmatrix} \quad (17)$$

Here, $\alpha_2 = \beta\alpha_1$. To make the tuning procedure simple, the value of β is kept as 0.6. Hence, there is only one tuning parameter (α_1). The value of this parameter (α_1) is selected by simulation. Table 4 gives the values of α_1 selected and the corresponding PID settings for different values of ε ($=\tau_d/\tau$). The settings are fitted by the following simple equations:

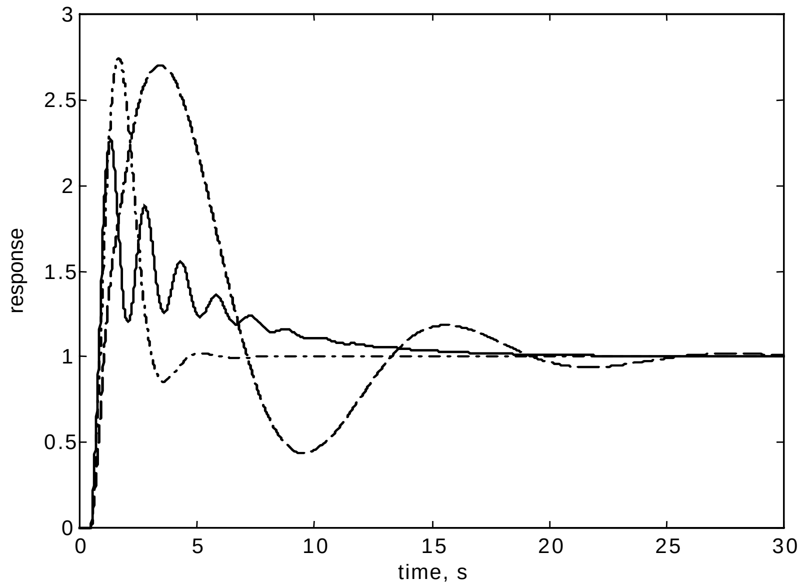

Fig. 4. Servo responses for unstable system ($k_p = 1$, $\tau_d = 0.5$, $\tau = 1$). Solid line: present method (Eq. (15)); dashed line: pole placement; dotted line: H–C method.

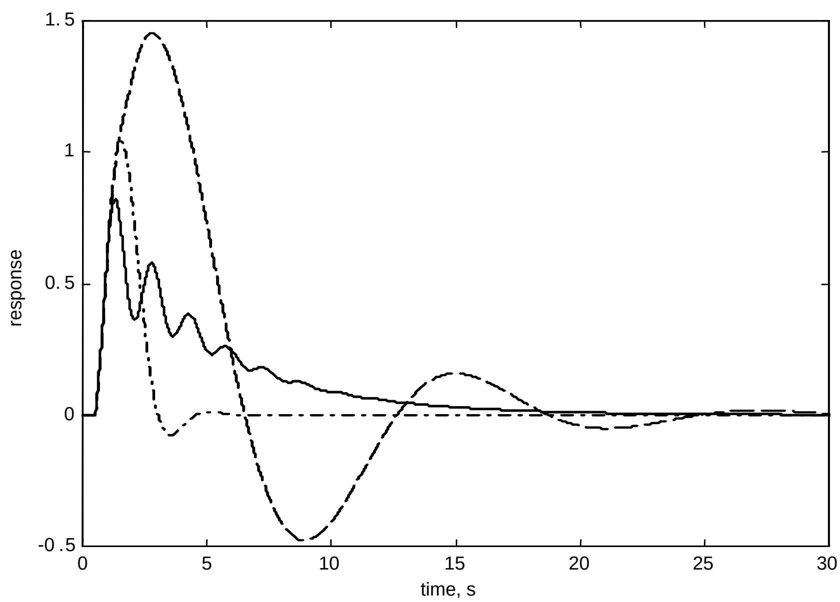

Fig. 5. Regulatory responses for unstable system ($k_p = 1$, $\tau_d = 0.5$, $\tau = 1$) with PID controller. Legend: as in Fig. 4.

$$
\begin{aligned}
k_c k_p &= 1.2824\varepsilon^{-0.8325} && \text{for } 0.01 \le \varepsilon \le 1.2 \\
\frac{\tau_I}{\tau} &= 5.5734\varepsilon - 0.0063 && \text{for } 0.01 \le \varepsilon \le 0.5 \\
\frac{\tau_I}{\tau} &= 0.483e^{3.3739\varepsilon} && \text{for } 0.5 \le \varepsilon \le 1.2 \\
\frac{\tau_D}{\tau} &= 0.507\varepsilon + 0.0028 && \text{for } 0.01 \le \varepsilon \le 1.2
\end{aligned} \qquad (18)
$$

The R^2 (regression coefficient) values for the above equations are 0.9942, 0.9993, 0.9853 and 0.9999, respectively.

Fig. 6 shows the servo responses of the closed loop system using the present settings, settings by Huang and Chen (1999), the IMC settings (Rotstein & Lewin, 1991) and that proposed by Visioli (2001). Fig. 6 shows the responses for $k_p = 1$, $\tau_d = 0.5$, $\tau = 0.5$. The filter time constant for the IMC method is selected by simulation as 1.5. The corresponding PID settings for the IMC method are $k_c = 2.444$, $\tau_I = 5.5$ and $\tau_D = 0.2386$ and for Visioli method are: $k_c = 2.4976$, $\tau_I = 2.8879$ and $\tau_D = 0.2901$. The present method gives a better performance than that of Huang and Chen

Table 4
Controller settings for unstable FOPTD systems using two tuning parameters (Eq. (17))

S. no	ε	α_1	$k_c k_p$	τ_I/τ	τ_D/τ
1	0.01	1.2	70.242	0.0601	0.0055
2	0.05	1.3	14.9935	0.2696	0.0271
3	0.1	1.4	7.9031	0.5385	0.0538
4	0.2	1.6	4.3109	1.1361	0.1062
5	0.3	1.85	3.1608	1.615	0.1567
6	0.4	2.1	2.5464	2.252	0.2071
7	0.5	2.4	2.19345	2.7792	0.2561
8	0.6	2.75	1.9633	3.2208	0.3040
9	0.7	3.05	1.7382	4.8	0.3561
10	0.8	3.4	1.5694	7.5336	0.4087
11	0.9	4.1	1.4220	10.5703	0.4627
12	1.0	4.6	1.3616	16.0206	0.5108
13	1.1	5.1	1.2946	20.900	0.5604
14	1.2	6.0	1.2439	23.8115	0.6087

(1999). However, the methods of IMC and Visioli give better performances. Fig. 7 shows the responses for the regulatory problem under perfect parameters. Since the method proposed by Visioli (2001) is based on minimization of ISE, this method gives the best performance (for ISE, IAE and ITAE values refer to Table 5a). Under perfect parameters values, method of Visioli gives the best performance. However, when 20% uncertainty is considered in the process gain (i.e., the controller is designed for $k_p = 1$, whereas in the process the process gain used is 1.2), the controller designed by Visioli (2001) could not stabilize the system (refer to Fig. 8). Similar stability problem arises for the Visioli method for an uncertainty of −20% in time constant (also refer to Table 6).

Let us consider a process with a larger time delay ($\tau_d = 0.8$). We get the PID settings for the present method as $k_c = 1.5694$, $\tau_I = 7.5336$ and $\tau_D = 0.4087$; for Huang and Chen method as: $k_c = 1.5419$, $\tau_I = 9.6573$ and $\tau_D = 0.2972$. The filter time constant for the IMC method is selected by simulation as 3.0. The corresponding PID settings for the IMC method are: $k_c = 1.7111$, $\tau_I = 15.4$ and $\tau_D = 0.3856$ and those for Visioli method are: $k_c = 1.6208$, $\tau_I = 3.6$ and $\tau_D = 0.4478$. Figs. 9 and 10 show the performance for the unstable system with $k_p = 1$, $\tau_d = 0.8$ and $\tau = 1.0$, respectively for a servo problem and for a regulatory problem. The performance of the present method is the best. Table 5b gives the comparison of ISE, IAE and ITAE values. When uncertainty in k_p (+20% uncertainty) is considered then both the Visioli method and the Huang and Chen method do not stabilize the system. Similar stability problem also arises when an uncertainty in τ_d (+20% uncertainty) is considered. The ISE values comparison under uncertainty in the model parameter is given in Table 7. Similar situation arises for system $k_p = 1$, $\tau_d = 1.1$ and $\tau = 1.0$. PID settings are calculated for this condition (for the present method as $k_c = 1.2946$, $\tau_I = 20.9$ and $\tau_D = 0.56$, for Huang and Chen method as: $k_c = 1.2325$, $\tau_I = 29.7549$ and $\tau_D = 0.4645$. The filter time constant for the IMC method is selected by simulation as 6.5. The corresponding PID settings for the IMC method are: $k_c = 1.3207$, $\tau_I = 55.8$ and $\tau_D = 0.5446$ and those for Visioli method are: $k_c = 1.2092$, $\tau_I = 4.1833$ and $\tau_D = 0.6687$). Visioli method does not stabilize the system. Figs. 11 and 12 show respectively the servo response and the regulatory response for ε $(=\tau_d/\tau) = 1.1$. The present method gives the best performance. The ISE, IAE and ITAE values are given in Table 5c.

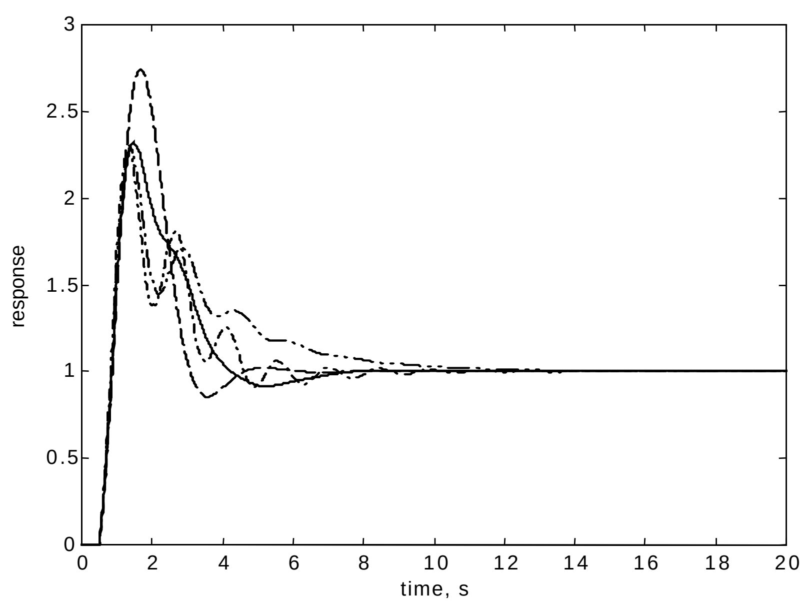

Fig. 6. Servo responses for unstable system ($k_p = 1$, $\tau_d = 0.5$, $\tau = 1$) with PID controller. Solid line: present method (Eq. (17)); dotted line: Visioli; dashed line: H–C method; dot-dashed line: IMC.

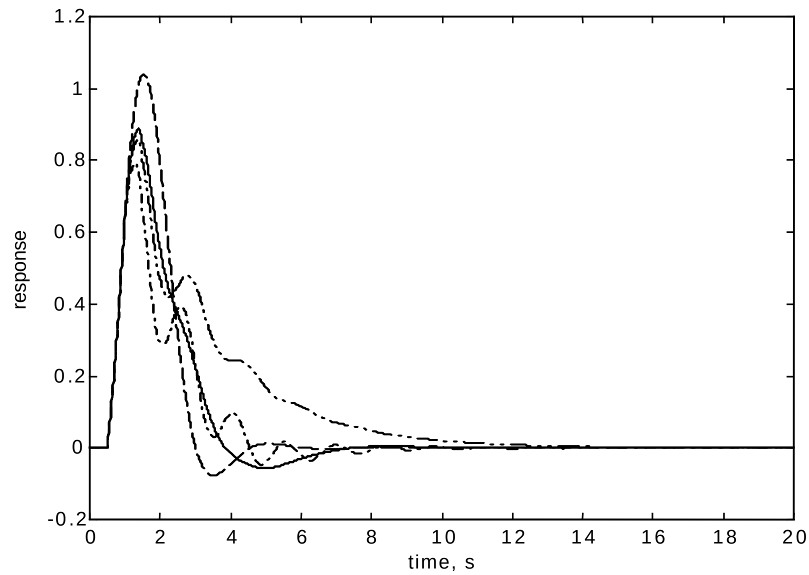

Fig. 7. Regulatory responses for unstable system ($k_p = 1$, $\tau_d = 0.5$, $\tau = 1$) with PID controller. Legend: as in Fig. 6.

Table 5
ISE, IAE and ITAE values for unstable systems with PID controller

Method	Servo problem			Regulatory problem		
	ISE	IAE	ITAE	ISE	IAE	ITAE
(a) When $k_p = 1$, $\tau = 1$, $\tau_d = 0.5$						
Present	2.5305	2.9173	5.5623	0.822	1.4896	3.1119
H–C	3.5209	3.0243	4.7927	1.1113	1.5026	2.6593
IMC	2.3963	3.6104	10.234	0.9781	2.2494	7.3194
Visioli	2.079	2.7322	5.4129	0.5758	1.2711	2.7487
(b) When $k_p = 1$, $\tau = 1$, $\tau_d = 0.8$						
Present	10.47	7.541	24.58	6.369	5.324	18.28
H–C	15.06	8.402	26.62	9.345	6.263	21.32
IMC	11.75	10.84	60.21	7.940	8.779	52.17
Visioli	9.340	6.846	23.99	4.168	4.124	15.76
(c) When $k_p = 1$, $\tau = 1$, $\tau_d = 1.1$						
Present	50.32	20.31	112.43	40.13	17.26	96.34
H–C	90.78	26.99	149.18	76.66	24.14	135.69
IMC	86.07	41.22	441.33	76.65	38.51	420.65

H–C: Huang and Chen (1999); IMC: Rotstein and Lewin (1991).
Visioli method gives unstable response for case c.

6. Design of PI controller for unstable systems

A PI controller is designed using the method discussed in the above section. Here, α_1 is tuned greater than 1 and α_2 is selected as $\beta\alpha_1$. The value of this parameter (α_1) and β is selected by simulation of the process model with PI controller. The value α_1 increases with the value of ε. The PID settings are fitted by the following simple equations:

$$k_c k_p = 0.8624\varepsilon^{-0.9744} \quad \text{for } 0.01 \le \varepsilon \le 0.6$$
$$\frac{\tau_I}{\tau} = 143.34\varepsilon^3 - 73.912\varepsilon^2 + 19.039\varepsilon - 0.2276 \quad \text{for } 0.01 \le \varepsilon \le 0.6 \qquad (19)$$

The R^2 (regression coefficients) values for the above equations are 0.9989 and 0.9967, respectively.

6.1. Simulation results for PI controllers

Let us consider a process ($k_p = 1$, $\tau = 1$, $\tau_d = 0.5$). We get the PI settings for the present method as $k_c = 1.7131$ and $\tau_I = 8.8542$, for Jung et al. (1999) method as: $k_c = 1.5353$ and $\tau_I = 7.5753$, for Poulin and Pomerleau (1996) method: $k_c = 1.7987$ and $\tau_I = 8.431$. Figs. 13 and 14 show the performance for the unstable system with $k_p = 1$, $\tau_d = 0.5$ and $\tau = 1.0$, respectively for a servo problem and for a

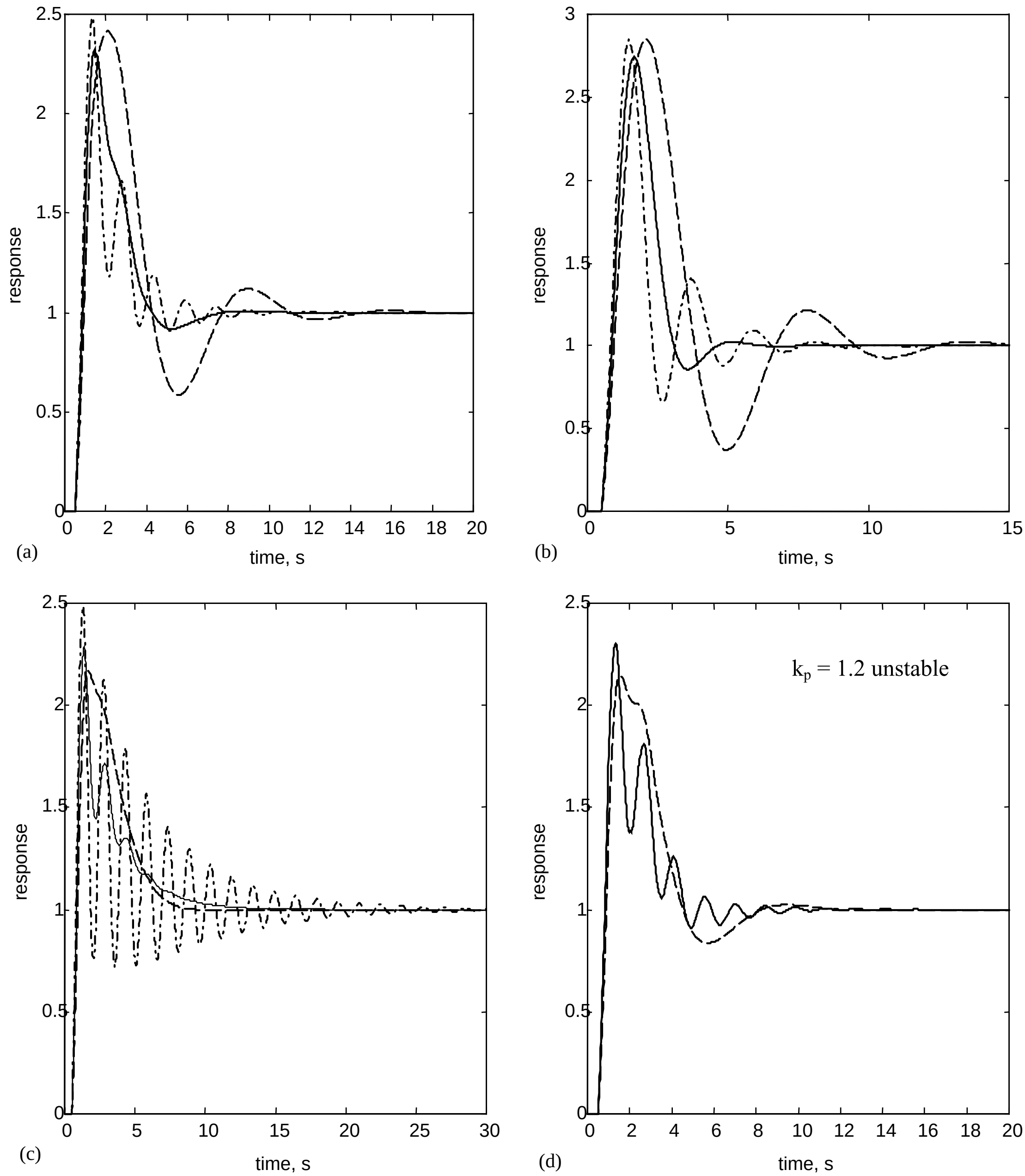

Fig. 8. Responses under uncertainty in k_p for unstable systems with PID controller: $k_p = 1.0$ for controller design; $k_p = 1.2$ or 0.8 in the process. (a) Present method (Eq. (17)); (b) H–C method; (c) IMC method; (d) Visioli method.

Table 6
ISE value comparisons for unstable systems for a PID controlled servo problem under parameter uncertainty (nominal parameters: $k_p = 1$, $\tau_d = 0.5$, $\tau = 1$)

S. no.	Method	ISE values for uncertainty in					
		20% k_p	-20% k_p	20% τ	-20% τ	+20% τ_d	-20% τ_d
1	Present	2.1498	4.335	2.5517	3.1172	4.2407	1.9381
2	H–C	3.1421	5.9912	3.2079	5.6877	11.5862	2.2955
3	IMC	3.0325	3.535	2.3963	8.484	4.7198	1.8811
4	Visioli	[a]	2.8767	1.9495	[a]	5.4122	1.521

H–C: Huang and Chen (1999); IMC: Rotstein and Lewin (1991).
[a] Means unstable.

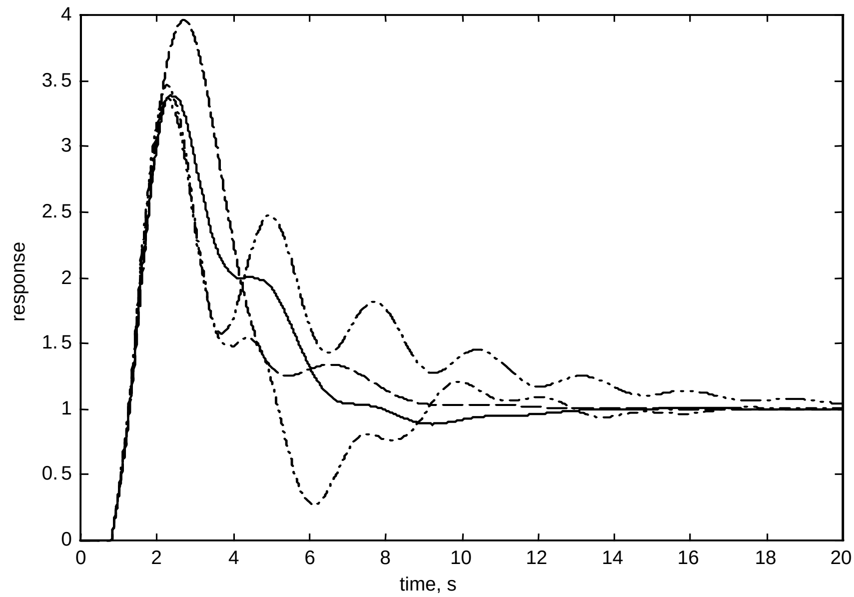

Fig. 9. Servo responses for unstable system ($k_p = 1$, $\tau_d = 0.8$, $\tau = 1.0$) with PID controller. Legend: as in Fig. 6.

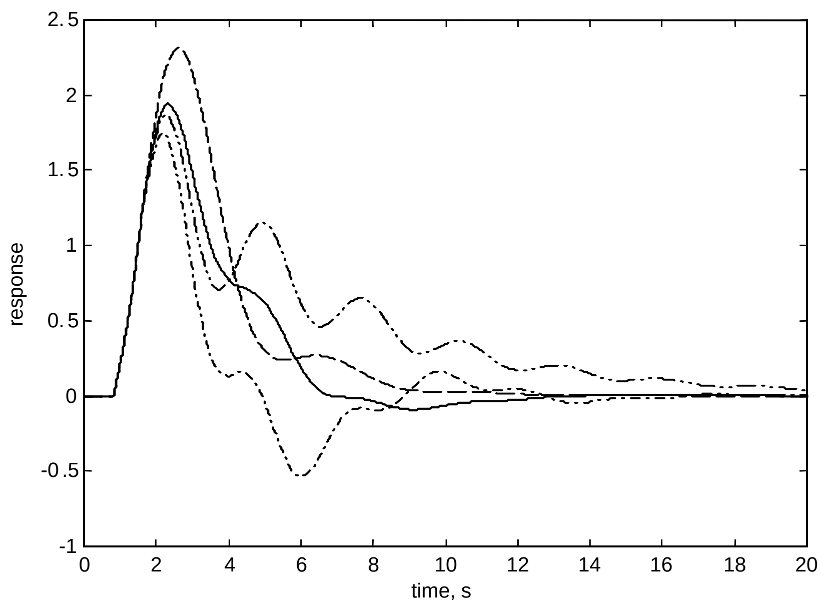

Fig. 10. Regulatory responses for unstable system ($k_p = 1$, $\tau_d = 0.8$, $\tau = 1.0$) with PID Controller. Legend: as in Fig. 6.

Table 7
ISE value comparisons for unstable systems for a PID controlled servo problem under parametric uncertainty for $\tau_d = 0.8$

S. no.	Method	ISE values for uncertainty in					
		20% k_p	−20% k_p	20% τ	−20% τ	+20% τ_d	−20% τ_d
1	Present	16.518	33.75	9.99	[a]	37.33	7.98
2	H–C	–[a]	37.37	8.39	[a]	[a]	10.37
3	IMC	15.503	51.69	13.13	[a]	[a]	9.14
4	Visioli	–[a]	24.31	16.43	[a]	[a]	5.68

H–C: Huang and Chen (1999); IMC: Rotstein and Lewin (1991).
[a] Means unstable.

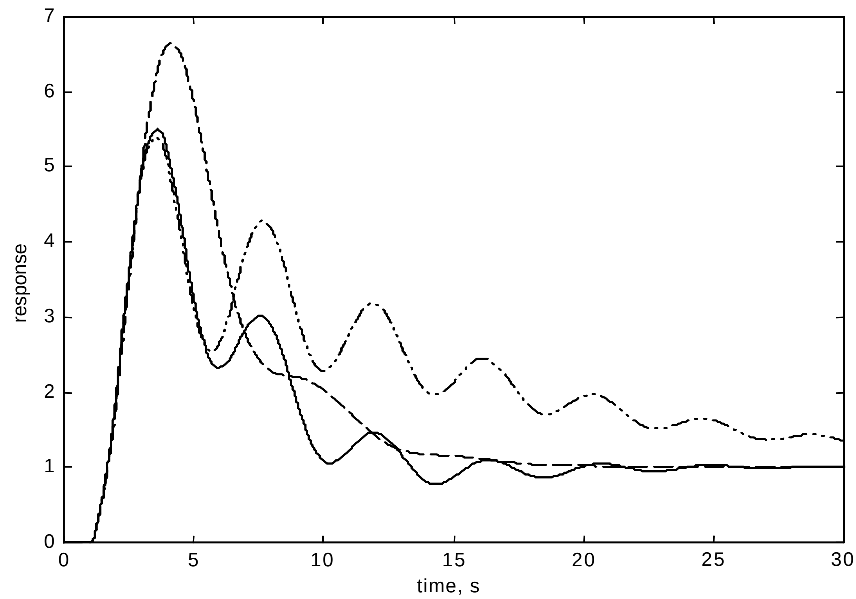

Fig. 11. Servo response for the unstable system ($k_p = 1$, $\tau = 1$, $\tau_d = 1.1$) with PID controller. Solid line: present method (Eq. (17)), dashed line: H–C; dot-dashed line: IMC (Visioli method is unstable).

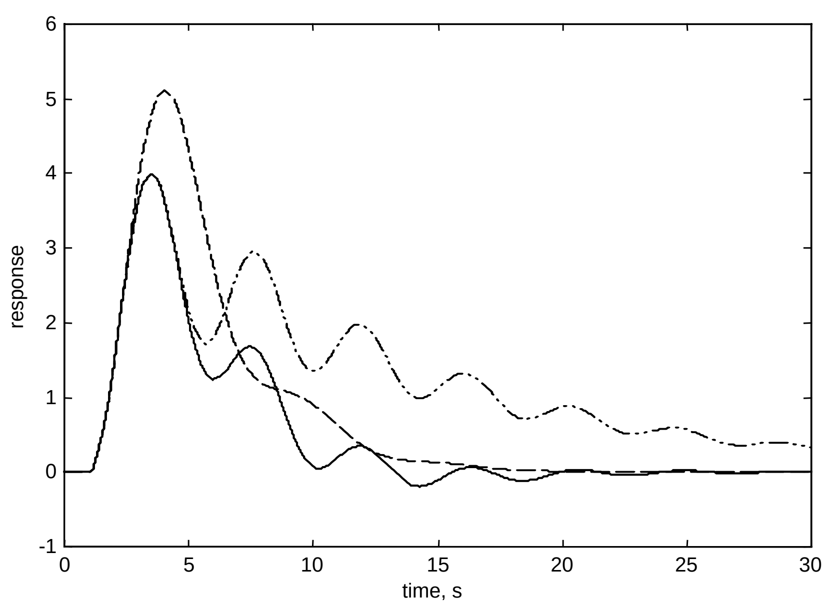

Fig. 12. Regulatory response for the unstable system ($k_p = 1$, $\tau = 1$, $\tau_d = 1.1$) with PID Controller. Legend: as in Fig. 11 (Visioli method is unstable).

regulatory problem. In the work of Jung et al. (1999), a set point filter is used. Here for fair comparison, the response is compared with out set point filter. The performance of the present method is the better than the method of Jung et al. (1999) both for the perfect parameter and under uncertainty in the model parameters. The ISE, IAE, ITAE values are reported in Table 8. The ISE values comparison under uncertainty in the model parameter is given in Table 9. Stability and robustness analyses are carried out in the next section.

7. Stability analysis

The stability of the given controllers can be checked by calculating gain and phase margin. Ho and Xu (1998) gave a method of calculating phase margin and gain margin of the unstable FOPTD system with PI controller. They have suggested that PI settings, which give larger phase margin, are preferred. In the present work, the method is extended for calculating phase margin and gain margin for the unstable FOPTD system with PID controller.

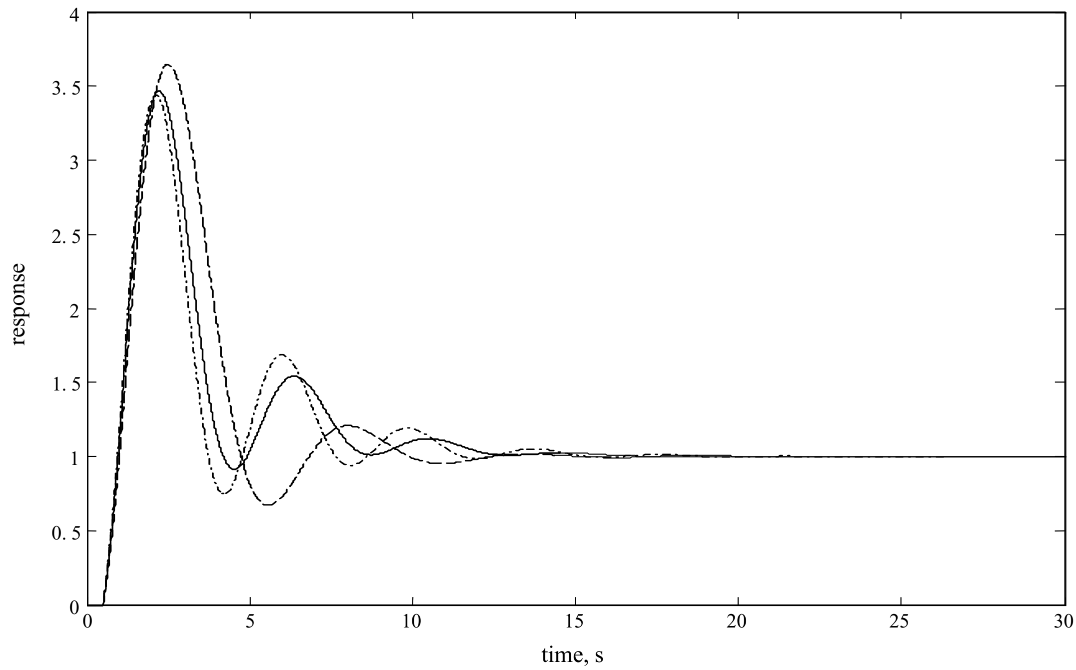

Fig. 13. Servo response of the system: $e^{-0.5s}/(s-1)$ with PI controller. Solid line: Present method with two tuning parameters; dotted line: Poulin–Pomerleau method; dashed line: Jung et al. method.

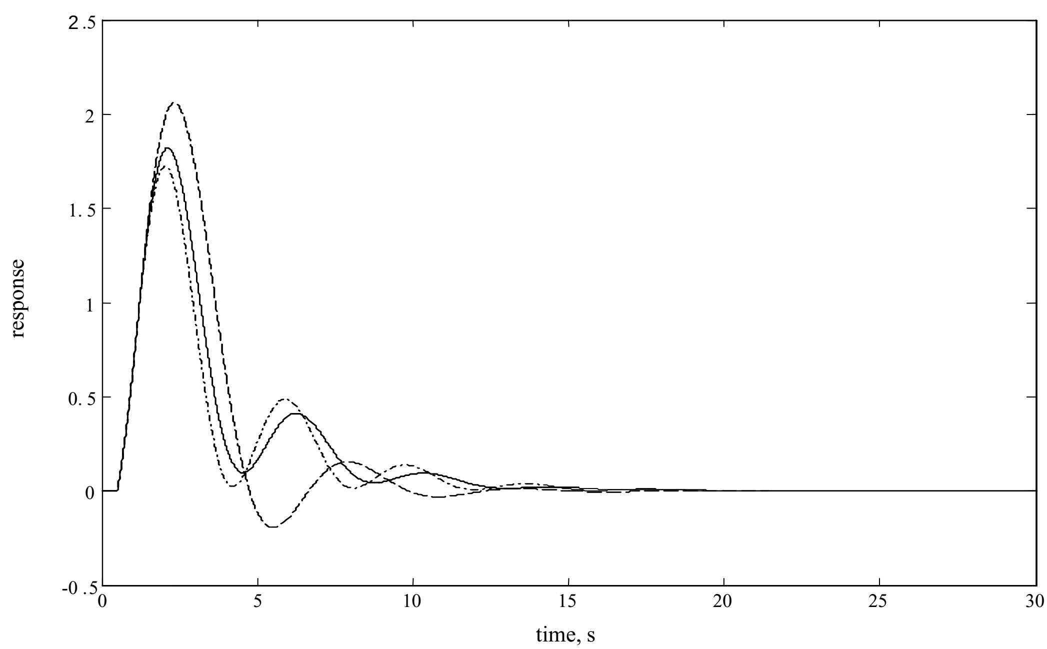

Fig. 14. Regulatory response of the system: $e^{-0.5s}/(s-1)$ with PI controller. Legend: as in Fig. 13.

Table 8
ISE, IAE and ITAE values for unstable systems ($k_p = 1$, $\tau = 1$, $\tau_d = 0.5$) using PI controller

Method	Servo problem			Regulatory problem		
	ISE	IAE	ITAE	ISE	IAE	ITAE
Present	9.59	6.74	22.64	5.50	5.168	19.09
Jung et al.	12.56	7.48	22.79	7.69	5.497	16.82
P–P	8.88	6.58	22.74	4.65	4.687	17.58

P–P: Poulin and Pomerleau (1996) and Jung et al. (1999).

Table 9
ISE value comparisons for unstable systems for PI controlled a servo problem under parametric uncertainty for $\tau_d = 0.5$

S. no.	Method	ISE values for uncertainty in					
		20% k_p	−20% k_p	20% τ	−20% τ	+20% τ_d	−20% τ_d
1	Present	9.07	20.22	7.88	[a]	37.13	6.71
2	Jung et al.	8.77	[a]	10.67	31.41	32.82	8.93
3	P–P	11.00	16.05	6.81	[a]	[a]	5.78

P–P: Poulin and Pomerleau (1996) and Jung et al. (1999).
[a] Means unstable.

Table 10
Gain margin and phase margin for the systems ($e^{-0.5s}/(s-1)$ and $e^{-0.8s}/(s-1)$) with the PID controller

Method	Present		Huang and Chen		Visioli		IMC	
	$\tau_d = 0.5$	$\tau_d = 0.8$	$\tau_d = 0.5$	$\tau_d = 0.8$	$\tau_d = 0.5$	$\tau_d = 0.8$	$\tau_d = 0.5$	$\tau_d = 0.8$
A_m	1.4056	1.164	1.515	1.2788	1.1616	1.2169	1.2755	1.1253
ϕ_m (°)	76.97	42.45	46.85	30.58	50.58	47.74	41.50	54.60

The process and controller transfer functions are denoted by $G_p(s)$ and $G_c(s)$ respectively. The loop transfer function is

$$G_c(s)G_p(s) = \frac{k_c k_p (1 + \tau_I s + \tau_I \tau_D s^2) e^{-\tau_d s}}{s\tau_I(\tau s - 1)} \tag{20}$$

The frequency at which the Nyquist curve has a phase of $-\pi$ (phase cross over frequency, ω_p) is obtained by solving the following equation:

$$\left(\frac{\pi}{2}\right) - \tan^{-1}\left(\frac{\tau_I\omega_p}{\tau_I\tau_D\omega_p^2 - 1}\right) + \tan^{-1}(\tau\omega_p) - \tau_d\omega_p = 0 \quad \text{if } (1 - \tau_I\tau_D\omega_p^2) < 0$$
$$-\left(\frac{\pi}{2}\right) + \tan^{-1}\left(\frac{\tau_I\omega_p}{1 - \tau_I\tau_D\omega_p^2}\right) + \tan^{-1}(\tau\omega_p) - \tau_d\omega_p = 0 \quad \text{otherwise.} \tag{21}$$

The gain margin is obtained by the following equation:

$$A_m = \left(\frac{\tau_I\omega_p}{k_c k_p}\right)\left\{\frac{1 + \tau^2\omega_p^2}{(1 - \tau_I\tau_D\omega_p^2)^2 + 1}\right\}^{0.5} \tag{22}$$

The frequency at which the Nyquist curve has amplitude of 1 is known as gain cross over frequency (ω_g), is obtained by solving the following equation:

$$k_c k_p = \tau_I\omega_g\left\{\frac{1 + \tau^2\omega_g^2}{(1 - \tau_I\tau_D\omega_g^2)^2 + 1}\right\}^{0.5} \tag{23}$$

The phase margin is given by the following equation:

$$\phi_m = \left(\frac{\pi}{2}\right) - \tan^{-1}\left(\frac{\tau_I\omega_g}{\tau_I\tau_D\omega_g^2 - 1}\right) + \tan^{-1}(\tau\omega_g) - \tau_d\omega_g \quad \text{if } (1 - \tau_I\tau_D\omega_g^2) < 0$$
$$\phi_m = -\left(\frac{\pi}{2}\right) + \tan^{-1}\left(\frac{\tau_I\omega_g}{1 - \tau_I\tau_D\omega_g^2}\right) + \tan^{-1}(\tau\omega_g) - \tau_d\omega_g \quad \text{otherwise.} \tag{24}$$

In the present work, the phase margin and gain margin are calculated for the system $G_p(s) = e^{-0.5s}/(s-1)$ and $e^{-0.8s}/(s-1)$ with the controller designed by different methods and are listed in the Table 10. Controller designed by the present method gives large phase margin and hence more stable than other methods. These are also shown by simulation study of the closed loop response as discussed earlier.

8. Robustness analysis of the controller

A control system is said to be robust if the closed loop system is stable even when the model parameters of the actual process are different than that used for controller design. To compare the robustness of the different controller design methods, the range of uncertainty in each of the model parameters for which the controller is stable is to be calculated. The robustness of the closed loop system for the perturbation separately in time delay, time constant and process gain is analyzed theoretically by Kharitonov's method (Sinha, 1994). In this method, the stability of four equations formed from Kharitonov polynomials is to be checked. The characteristic equation of the system using second order Pade's approximation for delay is

$$P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 \tag{25}$$

where

$$a_0 = k_c k_p \quad (26)$$

$$a_1 = k_c k_p(\tau_I - 0.5\tau_d) - \tau_I \quad (27)$$

$$a_2 = k_c k_p(0.0833\tau_d^2 - 0.5\tau_d\tau_I + \tau_I\tau_D) + \tau\tau_I - 0.5\tau_d\tau_I \quad (28)$$

$$a_3 = k_c k_p(0.0833\tau_I\tau_d^2 - 0.5\tau_d\tau_I\tau_D) + 0.5\tau_d\tau\tau_I - 0.0833\tau_d^2\tau_I \quad (29)$$

$$a_4 = 0.0833k_c k_p\tau_I\tau_D\tau_d^2 + 0.0833\tau\tau_I\tau_d^2 \quad (30)$$

Kharitonov's equations for $a_i^- \le a_i \le a_i^+$ ($i = 0, 1, 2, 3, 4$) are given below, where a_i^- and a_i^+ are the lower bound and upper bound for a_i, respectively:

$$a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 = 0 \quad (31)$$

$$a_0^- + a_1^- s + a_2^+ + s^2 + a_3^+ s^3 + a_4^- s^4 = 0 \quad (32)$$

$$a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 = 0 \quad (33)$$

$$a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 = 0 \quad (34)$$

For fixed values of k_p and τ, a perturbation in time delay τ_d i.e., when $(\tau_d - \Delta\tau_d) \le \tau_d \le (\tau_d + \Delta\tau_d)$ is substituted in the above coefficients and Kharitonov's equations are checked for stability using Routh–Hurwitz method (Kharitonov, 1978; Sinha, 1994). Similarly perturbation in k_p (for fixed τ and τ_d) and τ (for fixed k_p and τ_d) is evaluated and the stability ranges are listed in Tables 11 and 12. Stability and robustness analysis for control of unstable with PI controllers is given in Table 13.

The stability region for the controller designed by the present method is found to be more than the controllers designed by Visioli (2001) and IMC method (1991). Thus, the controller designed by the present method is more robust than the other methods. These are also shown by simulation study of the closed loop response as discussed earlier.

Table 11
Stability region for k_p, τ and τ_d for the system ($e^{-0.5s}/(s-1)$) with the PID controller

Process parameter	Present method	H–C method	Visioli method	IMC method
k_p	±0.37	±0.28	±0.16	±0.29
τ	±0.24	±0.23	±0.11	±0.17
τ_d	±0.11	±0.11	±0.06	±0.08

Table 12
Stability region for k_p, τ and τ_d for the system ($e^{-0.8s}/(s-1)$) with the PID controller

Process parameter	Present method	H–C method	Visioli method	IMC method
k_p	±0.22	±0.22	±0.16	±0.13
τ	±0.15	±0.16	±0.12	±0.09
τ_d	±0.12	±0.11	±0.10	±0.07

Table 13
Gain margin, phase margin, stability region for k_p, τ and τ_d for the system ($e^{-0.5s}/(s-1)$) with the PI controller

Method	ϕ_m (°)	A_m	Process parameter		
			k_p	τ	τ_d
Present	9.7562	1.3963	±0.20	±0.15	±0.09
P–P	8.7854	1.3253	±0.19	±0.12	±0.07
Jung et al.	9.5302	1.5398	±0.17	±0.18	±0.12

P–P: Poulin and Pomerleau (1996) and Jung et al. (1999).

9. Design of PI controller for stable systems

In chemical industries, PI controllers are more common than PID controllers. In the present section, a PI controller is designed for stable FOPTD systems using two tuning parameters. The value of α_1 is selected as either equal to 1 or less than 1 and α_2 is tuned equal to $\beta\alpha_1$ where $\beta = 0.6$. PI settings are tuned for different values of ε using simulation and the setting are fitted by the following simple equations.

$$k_c k_p = 0.9719\varepsilon^{-0.8915} \quad \text{for } 0.1 \le \varepsilon \le 1$$

$$\frac{\tau_I}{\tau} = 10.59\varepsilon^2 - 2.3588\varepsilon + 0.8985 \quad \text{for } 0.1 \le \varepsilon \le 0.4$$

$$\frac{\tau_I}{\tau} = 0.7719\varepsilon^4 - 3.6608\varepsilon^3 + 6.5791\varepsilon^2 + 5.1652\varepsilon + 2.8059 \quad \text{for } 0.4 \le \varepsilon \le 1.5 \quad (35)$$

The R^2 values for the above equations are 0.9931, 0.9933 and 0.9983, respectively.

9.1. Simulation results

The performance of the proposed method with two tuning parameters is evaluated on the system: $e^{-s}/(s+1)$. We get the PI settings for the present method as $k_c = 0.9127$ and $\tau_I = 1.3304$, for Abbas (1997) method as: $k_c = 0.4182$ and $\tau_I = 1.5$, for Ziegler and Nichols (1942) method: $k_c = 0.9$ and $\tau_I = 3.33$. Figs. 15 and 16 show the performance for the stable system with $k_p = 1$, $\tau_d = 1$ and $\tau = 1.0$, respectively for a servo problem and for a regulatory problem. The performance of the present method is the best for both servo and regulatory problem. For servo problem, though the overshoot is large it has faster settling time compared to the other methods. The ISE, IAE, ITAE value comparisons are given in Table 14. The ISE values comparison under uncertainty in the model parameter is given in Table 15. The present method is more robust.

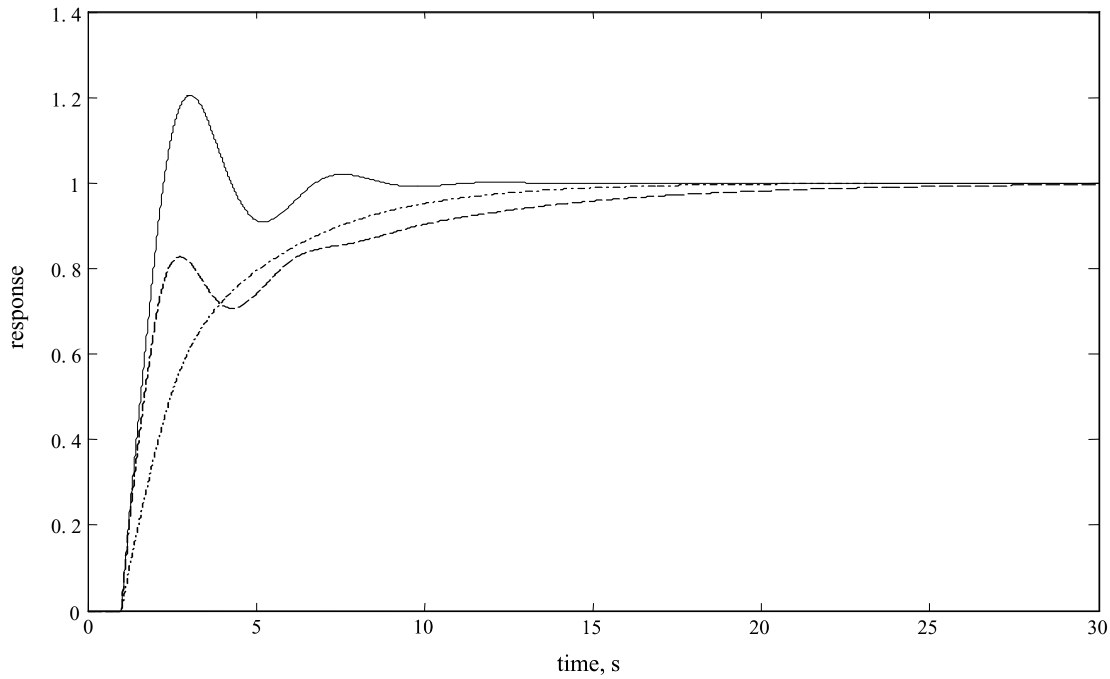

Fig. 15. Servo response of the system: $e^{-0.5s}/(s+1)$ with PI controller. Solid line: present method with two tuning parameters; dotted line: AA method; dashed line: Z–N method.

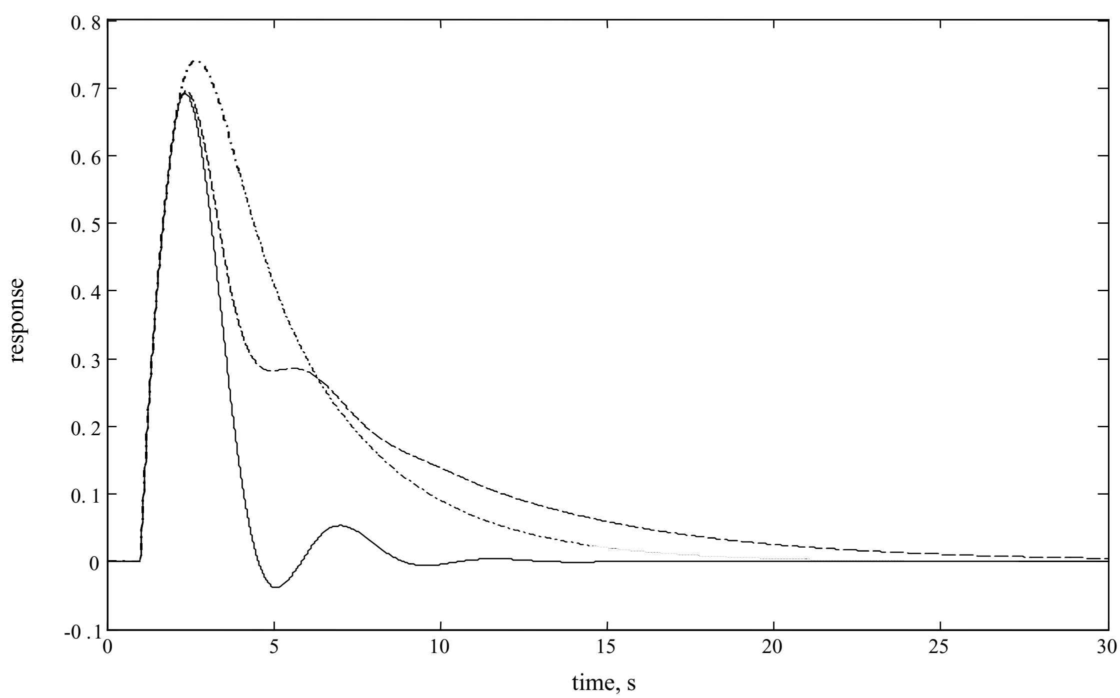

Fig. 16. Regulatory response of the system: $e^{-0.5s}/(s+1)$ with PI controller. Legend: as in Fig. 15.

10. Design of PID controller for stable systems

The method proposed (with out any tuning parameter) in the earlier section for the design of PID controllers for stable systems gives oscillatory responses though gives almost similar performance of controller designed by Z–N method. Hence, there is need to improve the method using two tuning parameters as discussed for unstable systems. In the present

Table 14
ISE, IAE and ITAE values for stable systems ($k_p = 1$, $\tau = 1$, $\tau_d = 0.5$) using PI controller

Method	Servo problem			Regulatory problem		
	ISE	IAE	ITAE	ISE	IAE	ITAE
Present	1.4447	2.037	3.288	0.7343	1.534	4.54
Abbas	2.1674	3.592	11.10	1.6281	3.586	18.26
Z–N	1.7784	3.685	17.94	1.228	3.672	25.04

Z–N: Ziegler and Nichols (1942) and Abbas (1997).

Table 15
ISE value comparisons for stable systems under parameter uncertainty using PI controller (parameters for controller design: $k_p = 1$, $\tau_d = 1$, $\tau = 1.0$)

Method	Servo			Regulatory		
	ISE values for uncertainty in			ISE values for uncertainty in		
	20% k_p	20% τ	20% L	20% k_p	20% τ	20% L
Present	1.5072	1.502	1.7796	0.9649	0.7075	0.919
Z–N	1.5877	1.8254	1.927	1.426	1.2015	1.318
Abbas	1.8978	2.2538	2.3	1.937	1.6204	1.732

Z–N: Ziegler–Nichols method.

section, a PID controller is designed for stable FOPTD systems using two tuning parameters. The value of α_1 is selected as equal to 1.2 and α_2 is tuned equal to $\beta\alpha_1$ where $\beta = 0.8$. PID settings are tuned for different values of ε using simulation and the setting are fitted by the following simple equations:

$$\begin{aligned} k_c k_p &= 1.377\varepsilon^{-0.8422} && \text{for } 0.1 \le \varepsilon \le 1 \\ \frac{\tau_I}{\tau} &= 1.085\varepsilon^{0.4777} && \text{for } 0.1 \le \varepsilon \le 1 \\ \frac{\tau_D}{\tau} &= 0.3899\varepsilon + 0.0195 && \text{for } 0.1 \le \varepsilon \le 1 \end{aligned} \qquad (36)$$

The R^2 values for the above equations are 0.9974, 0.9958 and 0.9975, respectively.

10.1. Simulation results

The performance of the proposed method with two tuning parameters is evaluated on the system: $e^{-0.5s}/(s+1)$. We get the PID settings for the present method as $k_c = 2.398$ and $\tau_I = 0.7852$ and $\tau_D = 0.2206$. The performance of the controller is compared with the Z–N method and IMC method (Wang et al., 2001). Figs. 17 and 18 show the performance for the stable system with $k_p = 1$, $\tau_d = 0.5$ and $\tau = 1.0$, respectively for a servo problem and for a regulatory problem. The performance of the present method is the better than that of the Z–N method and IMC method for regulatory problem and there are no oscillations in the response for servo problem. For servo problem, though the overshoot is large it has faster settling time compared to the other methods. The ISE, IAE, ITAE value comparisons are given in Table 1. The ISE values comparison under uncertainty in the model parameter is given in Table 2.

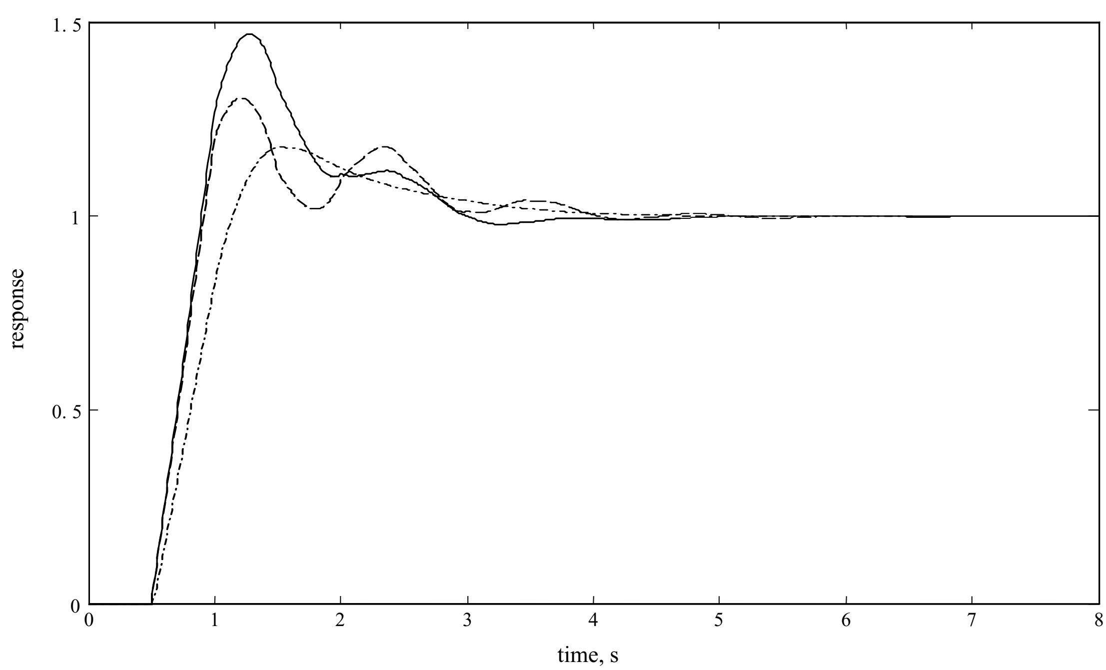

Fig. 17. Servo response of the system: $e^{-0.5s}/(s+1)$ with PID controller. Solid line: Present method with two tuning parameters; dotted line: IMC method; dashed line: Z–N method.

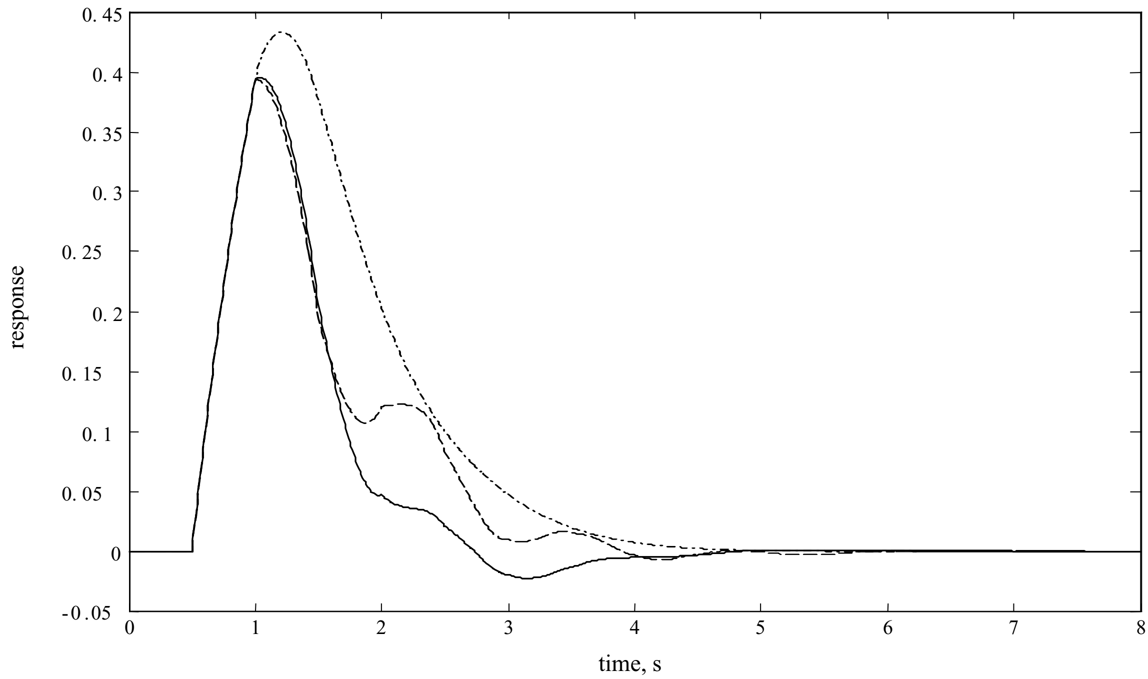

Fig. 18. Regulatory response of the system: $e^{-0.5s}/(s+1)$ with PID controller. Legend: as in Fig. 17.

11. Conclusions

A simple method is proposed for PI/PID controller settings for stable FOPTD and also for unstable FOPTD systems. The method gives a simple set of equations for the controller settings. The present method is robust for uncertainty in the model parameters. For stable systems, the present method gives the best performance when compared to that of Ziegler and Nichols (1942) method and that by Abbas (1997). For unstable systems, the performance of the present PID controller is significantly better than the method proposed by Huang and Chen (1997) and Visioli (2001) and IMC method and the response of proposed PI controller is better when compared to that of Jung et al. (1999). For unstable systems also, robust responses are obtained. The stability and robustness analysis of the proposed controller are evaluated theoretically by phase margin method and Kharitonov theorem respectively.

References

Abbas, A. (1997). A new set of controller tuning relations. *ISA Transactions, 36*, 183.

Astrom, K. J., & Hagglund, T. (1995). *PID controller: Theory, design and tuning*. Triangle Park, NC: ISA Publication Research.

Chandrashekar, R., Padmasree, R., & Chidambaram, M. (2002). Design of PID controllers for unstable systems with delay by synthesis method. *Indian Chemical Engineering, 44*(2), 82–88.

Cheng, S. L., & Hwang, C. (1998). Designing PID controllers with a minimum IAE criterion by a differential evolution algorithm. *Chemical Engineering Communications, 170*, 83–115.

Chidambaram, M. (1997). Control of unstable systems: A review. *Journal of Energy, Heat and Mass Transfer, 19*, 49.

Chidambaram, M. (1998). *Applied process control*. New Delhi: Allied Publishers.

Clement, V. C., & Chidambaram, M. (1997a). PID control of stable first-order plus time delay system. *Indian Chemical Engineering, 39*, 9.

Clement, V. C., & Chidambaram, M. (1997b). PID control of unstable FOPTD systems. *Chemical Engineering Communications, 162*, 63.

Cohen, G. H., & Coon, G. A. (1953). Theoretical investigation of retarded control. *Transactions of the ASME, 75*, 827.

DePaor, A. M., & O'Malley, M. (1989). Controllers of Ziegler–Nichols type for unstable process with time delay. *International Journal of Control, 49*, 1273.

Haalman, A. (1965). Adjusting controllers for dead time processes. *Control Engineering, 12*, 71.

Ho, W. K., & Xu, W. (1998). PID tuning for unstable processes based on gain and phase margin specifications. *IEE Proceedings on the CTA, 145*, 392.

Huang, H. P., & Chen, C. C. (1997). Control system synthesis for open loop unstable processes with delay. *IEE Proceedings on the CTA, 144*, 334.

Huang, H. P., & Chen, C. C. (1999). Auto-tuning of PID controllers for second order unstable systems having dead time. *Journal of Chemical Engineering (Japan), 32*, 579.

Jung, C. S., Song, H. K., & Hyun, J. C. (1999). A direct synthesis method of unstable first-order time delay processes. *Journal of Process Control, 9*, 265.

Manoj, K. J., & Chidambaram, M. (2001). PID controller tuning for unstable systems by optimization method. *Chemical Engineering Communications, 185*, 91.

Marchetti, G., Scali, C., & Lewin, D.R. (2001). Identification and control of open loop unstable processes by relay methods. *Automatica, 37*, 2049.

Poulin, E., & Pomerleau, A. (1996). PID tuning for integrating and unstable processes. *IEE Proceedings on the CTA, 143*, 429.

Rivera, D. E., Morari, M., & Skogetad, S. (1986). IMC-PID controller design. *Industrial Engineering and Chemical Process, Design and Development, 25*, 252.

Rotstein, G. E., & Lewin, D. R. (1991). Simple PI and PID type controllers for unstable systems. *Industrial Engineering and Chemical Research, 30*, 1964.

Sinha, N. K. (1994). *Control systems* (pp. 142–144). India: Weily Eastern Ltd.

Smith, C. A., & Corripio, A. B. (1985). *Principles and practice of automatic process control*. New York: McGraw Hill.

Venkatasubramaniam, V., & Chidambaram, M. (1994). Design of P and PI controllers for unstable FOPTD model. *International Journal of Control, 60*, 137.

Visioli, A. (2001). Optimum tuning of PID controllers for integrating and unstable processes. *IEE Proceedings on the CTA, 148*, 180.

Wang, Q.-C., Hang, C. C., & Yang, X.-P. (2001). Single loop controller design via IMC principles. *Automatica, 37*, 2041.

Ziegler, J. G., & Nichols, N. B. (1942). Optimum settings for automatic controllers. *ASME Transactions, 64*, 759.